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A METHOD FOR OBTAINING THE PARAMETERS OF ELECTRON-DENSITY PROFILES FROM TOPSIDE IONOGRAMS

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U. S. DEPARTMENT OF COMMERCE
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A METHOD FOR OBTAINING THE PARAMETERS OF ELECTRON DENSITY PROFILES FROM TOPSIDE IONOGRAMS

Robert S. Lawrence and Margot J. Hallenbeck

We present sample results of a model-fitting procedure which involves adjustment of physically meaningful parameters of a model ionosphere so as to minimize the mean square difference between the observed virtual depths and those calculated for the model. The adjustment is made subject to additional constraints such as (1) physically reasonable limits to the values of the parameters and (2) agreement with any available auxiliary information such as the electron density at the satellite. The method requires more computer time than do conventional methods but compensates by making extremely economical use of observational data. It has been demonstrated that the method will provide useful information from data which are too scanty to be usable by lamination or polynomial methods. When, as occasionally happens, the process fails to find a useful answer, the reason appears to be that the ionosphere cannot be represented by the model being used. On the other hand, the use of physical limits on the parameters seems to insure that erroneous or multiple solutions will not occur.

1. INTRODUCTION

Many methods are available for deriving information about the vertical distribution of electrons from sweep-frequency ionospheric soundings, One of the earliest of these, suggested by Appleton and Beynon [1940], involved the assumption that the profile of the ionosphere is approximately parabolic. With this assumption, three points on the ionogram were used to determine the parameters of the parabola. In fact, the electron distribution is much more complicated than a simple parabola, and for realistic results it is necessary to elaborate this simple procedure.

Modern attempts at elaboration lie between two limiting types of approach. One of these, the "polynomial" approach, involves the use of a polynomial of high order to represent the entire ionosphere [Titheridge, 1961]. The other, the "lamination" method [Budden, 1961], involves a large number of slabs, each of which is assumed to have a linear variation of plasma frequency. Between these two extremes there exists many lamination methods which involve the use of non-linear slabs.

Such methods, with various elaborations to surmount practical difficulties, have been used quite successfuly to produce large numbers of useful electron-density profiles below the peak of the F region [Thomas et al., 1958; Wright et al., 1963]. With the advent of the first topside ionograms from the satellite-borne sounder, Alouette (1962 Beta Alpha One), it was only natural to use the same procedures to obtain topside profiles. Nelms [1963], Thomas et al. [1963], Doupnik [1963], Fitzenreiter and Blumle [1964], and Paul and Wright [1964], among others, have prepared computer programs for this purpose. These programs produce satisfactory topside profiles from the Alouette soundings and they have been widely used.

The primary purpose for producing ionospheric profiles is, of course, to learn more about the physical conditions existing in the upper atmosphere, especially the involved processes of the formation, loss, and movement of the ions and free electrons. For example, at heights well above the F peak, the plasma scale height is determined by the slope of a semi-logarithmic plot of electron density. If the ionic composition is known, the plasma scale height gives a direct measurement of temperature. This type of reasoning has been carried several steps further. Rishbeth (see Calvert et al., 1964) for example, has compared topside profiles, deduced from ionograms by a lamination method, with a number of theoretical profiles based upon various assumptions about the temperature and ionic composition. In this way, he succeeded in determining values for several parameters of his model at heights well above the F peak where production and loss processes could be neglected.

The success of Rishbeth's approach using a small number of adjustable parameters led us to consider the possibility of mechanizing his method so that, with a computer, more data could be analyzed and more parameters could be adjusted with greater ease and objectivity. If a computer program to fit a model to a known profile is feasible, it is, conceptually, only a small step to consider a still more complex computer program which fits the parameters of the model directly to the ionogram. Both of these computer programs have been written and they operate successfully. It is the purpose of this paper to describe the methods used by these programs and to present sample results.

There is a compelling reason for taking the second, and larger, of these two steps, thereby replacing the conventional determination of an electron-density profile. The conventional methods require observation of the virtual depth at a large number of frequencies; large, that is, compared to the number of parameters we hope to adjust. They are generally ill adapted to the use of incomplete ionograms or to the simultaneous use of bits and pieces of the ordinary and extraordinary traces and of the ground-reflected trace. Therefore, in the case of incomplete ionograms or data from the fixed-frequency topside sounder, 1964-51A (S-48) [Knecht et al., 1962], the conventional methods are frequently quite incapable of producing accurate results.

To recapitulate, we are presenting a method of analysis which is applicable to cases where the ionospheric profile is reasonably smooth and which is particularly valuable for those cases where the number of data points is limited or some of the values are imprecisely known. Because of the complicated structure of the lower ionosphere, the method is not recommended for use with ground-based ionograms.

In this paper we illustrate our method with a particular model (Rishbeth's) for the upper ionosphere, a particular method of calculating virtual depths (VIRDEP), and a particular minimization process (MINMYZ). However, the reader must keep in mind that the success of the method is not based upon these particulars but rather upon the incorporation of non-rigid bounds on the individual variable parameters and upon the use of available constraints on the model as a whole.

2. DESCRIPTION OF THE COMPUTER PROGRAM

The computer program is given: (1) an ionospheric model involving several adjustable parameters; (2) an initial set of parameters with their individual bounds; (3) the frequencies, modes (ordinary or extraordinary), and observed virtual depths of several pulses reflected back to a topside sounder; (4) any known information about the ionosphere which may serve as a constraint; and (5) essential information at the sounder - height, geomagnetic dip, and gyrofrequency. The program must then adjust the parameters of the model so that the theoretical virtual depths calculated from the model will best agree with the observed virtual depths. Implicit in this statement is the assumption that there is a unique solution which gives "best" agreement. The problem seems to the authors far too complicated to permit a mathematical demonstration of uniqueness, but experience suggests that, for practical cases, if a solution exists which gives reasonable agreement it is unique. We define best agreement as that which minimizes the mean square of the residuals.

From this statement of the problem we can already see two of the essential ingredients of the program. First, there is a subroutine, VIRDEP, that computes the virtual depth to be expected for a pulse of arbitrary frequency reflected from the model ionosphere. Second, there is a subroutine, MINMYZ, that tries to adjust a set of parameters in such a way as to minimize the sum of the squares of an arbitrary set of residuals which result from the use of those parameters. These subroutines are described later in some detail.

Another essential ingredient of any practical program of this nature is a procedure for accepting bounds on the parameters and for accepting other constraints on the model. The parameters of the model are intended to have at least a modicum of physical meaning. We are not interested in any narrowly empirical solution which involves such unpalatable features as a negative amount of some atmospheric constituent, and we do not want the computer to spend time searching for such a solution. If we specify physically realistic bounds on the values which the parameters may take, we can ensure that the computer will find only acceptable solutions.

In addition, it is highly desirable to be able to include, as constraints, any pertinent information which might be available from independent sources. For example, the critical (penetration) frequency of the ionosphere might be known from ground-based observations, or the total electron content might be known from Faraday-rotation measurements of the satellite signal. Such information comprises a constraint on a function of several parameters rather than a bound on the value of any one parameter. The methods of handling bounds and constraints are detailed in a later paragraph.

General operation The program must be given a set of initial values of the parameters. Its first task is to check to see whether these initial values satisfy all the constraints. Generally, they will fall within the specified bounds, but they may violate a constraint such as a known critical frequency or a known electron density at the sounder. If one of these constrains is violated, the program uses the subroutine MINMYZ to adjust the parameters until all constraints and bounds are satisfied. This process, which is relatively simple, reduces the amount of adjustment which will be needed later when virtual depths will have to be calculated laboriously for each trial model.

After a suitable set of parameters is obtained, the program uses MINMYZ again, this time trying to adjust the parameters so that the virtual depths calculated by VIRDEP will nearly agree with the observations and the parameters will continue to satisfy all bounds and constraints. If the actual ionosphere differs radically from any acceptable configuration of the model, there will be no solution. Experience suggests that when a solution is found it is unique, i.e., it does not depend upon the initial values assumed for the parameters. Failure to find a solution appears to indicate incompatibility between the data and the model. Provision is made for printing the current values of the parameters, the trial virtual depths, and a graph of the trial model profile as frequently as desired to check progress. Naturally, this information is always presented for the final result.

The virtual depth subroutine, VIRDEP The virtual, or apparent, depth of reflection of a pulse from a topside sounder is

$$d = \int_{h_r}^{h_s} \mu' \, dh, \qquad (1)$$

where h_r is the height of reflection of the pulse; h_s is the height of the sounder; and μ', the group refractive index of the ionosphere, is a complicated function of the sounding frequency, the electron density and the geomagnetic field. VIRDEP uses the formulation presented by Shinn and Whale [1952], assuming that the propagation is vertical and that the dip angle of the magnetic field is independent of height. VIRDEP assumes that the strength of the magnetic field varies inversely as the cube of the distance from the center of the earth.

Before integrating equation (1), VIRDEP must find h_r. Since our model is monotonic above h_{max}, h_r can be determined by an ordinary binary search.

VIRDEP divides the range of integration into several slabs of thickness averaging about 100 km, the ones near the sounder being the thickest. In order to achieve an accuracy of one kilometer, 8-point Gaussian integration is required for each slab. The lowest slab presents a special problem because μ' becomes infinite at h_r. The bottom kilometer of this slab is evaluated by an analytic integration which assumes a linear variation of electron density. In this case μ' is approximated by the first of two terms of a series expansion given by Budden [1961].

The minimization process, MINMYZ MINMYZ is a gradient-projection subroutine developed by Slutz and Winkelman [1964]. Its purpose is to adjust a set of variables so as to minimize an arbitrary function of them. In the present application, the variables are the parameters of the model ionosphere and the function to be minimized involves the mean square difference between the observed virtual depths and the corresponding depths calculated by VIRDEP.

Given m adjustable parameters P_i and a procedure for computing n ($\geq m$) errors (such as discrepancies in virtual depth) E_j, MINMYZ first evaluates numerically the partial derivatives $\partial E_j/\partial P_i$ for $j \leq n$ and all $i \leq m$. This is done by one-sided rather than central differencing in order to reduce from $2mn$ to $mn+1$ the number of times VIRDEP must be used.

In geometrical terms, the operation of MINMYZ may be described as follows. From the array of partial derivatives, MINMYZ determines the gradient of the error surface in $(m+1)$-dimensional space and from the individual derivatives estimates the distance which that gradient should be projected to find a minimum. The step is taken and VIRDEP is used n times to assess the result. Comparison of the actual changes in the errors with those which were expected permits MINMYZ to decide whether it will be profitable to take additional steps in the same direction. If so, they are taken, using VIRDEP n times to check the result of each step. MINMYZ continues until no further improvement can be made locally in that direction. The program may use MINMYZ several times to get new gradient directions.

When MINMYZ finds a minimum there is naturally no assurance that it is the global minimum. It is the responsibility of the user to design the function which defines the errors so that there will be few, if any, local minima in which to become trapped. Our procedure for doing this includes the use of physical constraints and bounds on the parameters. These are described below. In addition, we have found it necessary to invent artificial definitions of the error for those cases where, in the trial ionosphere, the probing frequency cannot propagate in the vicinity of the sounder or where it penetrates all the way to the ground. Unless something special is done with these cases, the partial derivatives are zero and MINMYZ receives no indication of which way to go.

The accommodation of bounds and constraints At first glance, the present method seems to fall in the category of problems commonly faced in operations research. Nonlinear programming involves the minimization (or maximization) of a given function of several variables while simultaneously satisfying a set of linear or nonlinear constraints on those variables. An enormous effort has been devoted to the solution of this economically important problem [see for example, Zoutendijk, 1960; or Rosen, 1961], and it is tempting to try to use here the results of that work. This seems to be a vain hope because of important differences in emphasis. The typical problem in operations research involves, perhaps, hundreds of variables, and the function to be minimized is a simple function of those variables. Furthermore, the bounds on the variables are absolutely rigid.

As an example, the number of oil refineries which can be used to produce a specific grade of fuel cannot be negative nor can it exceed the total number available. Rigid bounds cause abrupt changes in the direction of progress toward a minimum and so may increase the number of calculations of the function to be minimized.

Our problem, on the other hand, involves a small number of variables (the parameters of the model), but the function to be minimized is extremely complicated. Its calculation involves many lengthy numerical integrations of the group refractive index. The refractive index is an involved function of the plasma frequency which, in turn, is a complicated function of the parameters. The bounds on our parameters are not, by nature, absolutely rigid. To be sure, the temperature cannot be negative, but we can give more information than that. We know that it probably lies between 500°K and 1500°K and we can specify those values as bounds, although we would be just as satisfied with an answer of 499° as with 501°.

Thus, we have chosen to ignore the absolutely bounded methods of operations research and to include our bounds and constraints in the error function to be minimized. In this way we tend to avoid an excessive number of calculations of our complicated error function. Each bound or constraint is expressed in terms of a function which can be treated in exactly the same way as a residual in virtual depth. In essence all errors are "normalized" to a virtual depth residual. For example, we have defined the "error", E_T, connected with temperature T as

$$E_T = 30 \left(\frac{1000 - T}{500}\right)^3 \qquad (2)$$

so that temperatures of 500° or 1500° would have the same effect on MINMYZ as would a virtual-depth residual of 30 km, and temperatures venturing outside this range would rapidly become less acceptable. Likewise, an "error" of similar form is used for any of the available constraints. In the case of a constraint upon the critical frequency, we might require that a computed frequency, f_c be within 0.1 Mc/s of the given frequency, f_g. The error would then be defined as

$$E_f = 10 \frac{(f_c - f_g)^3}{(0.1)^3}. \qquad (3)$$

The inclusion of these "errors" defined upon the variable parameters, themselves, and upon the given constraints lends much to the success of the method. It not only permits the incorporation of any extra information (constraints) into the final results, but it also helps to guarantee an acceptable solution by keeping the parameters within physically reasonable bounds.

The final function to be minimized is the mean square value of all such errors and of all residuals between the observed and computed virtual depths. We describe the degree of failure of the program to make a perfect fit to the observations by evaluating the estimated rms error,

$$s = \left[\frac{1}{n - k}\sum_{i=1}^{n} r_i^2\right]^{\frac{1}{2}}, \qquad (4)$$

where r_i are the residuals in virtual depth, n is the number of observed virtual depths, and k is the number of independently adjustable parameters, i.e., the number of adjustable parameters reduced by the number of constraints and further reduced by one to allow approximately for the fact that all parameters are bounded.

Sample results We now illustrate our method with several examples which were run on the IBM 7090 computer. The model used in our examples involves the following adjustable parameters:

h_a - the height at which the concentration of O^+ equals the concentration of He^+ (km)

σ - the ratio of concentration of H^+ and He^+ at height h_a

N_a - the concentration of electrons at height h_a (cm^{-3})

T - the electron temperature ($^\circ K$)

h_{max} - the height of maximum electron density (km).

The electron density N_h at height h (km) is calculated from the parameters in the following way:

$$\left[\frac{N_h}{N_a}\right]^2 = \frac{e^{-Z_h - e^{Z_m - Z_h}} + e^{-Z_h/4} + \sigma e^{-Z_h/16}}{2 + \sigma}, \qquad (5)$$

where

$$Z_h = \frac{K\,(h - h_a)}{T\,(R_o + h_a)(R_o + h)}$$

$$Z_m = \frac{K(h_{max} - h_a)}{T\,(R_o + h_a)(R_o + h_{max})}$$

$$R_o = 6370 \text{ km}$$

$$K = 765.826 \times 10^6 \ ^\circ\text{K km.}$$

We do not intend to try to justify this particular model, but only to illustrate our method with its use. The method should work as well for any reasonable model involving a similar number of parameters.

In the sample computer runs, functions of the form

$$P_n \left(\frac{P_m - P_c}{P_r}\right)^n \tag{6}$$

were used as "errors" for MINMYZ in order to establish the following upper and lower bounds on the parameters:

$$500^\circ \text{ K} < T < 1500^\circ \text{ K}$$

$$0 < \sigma < 1.25$$

$$375 \text{ km} < h_a < 900 \text{ km}$$

$$.5 \times 10^4 \text{ cm}^{-3} < N_a < 5.5 \times 10^4 \text{ cm}^{-3}$$

$$100 \text{ km} < h_{max} < 400 \text{ km.}$$

The P_m is the mean value of the desired upper and lower bounds for each parameter, P_c is the current value of the parameter, P_r is the half-range of permitted variation for the parameter and is simply the difference between the upper bound and P_m, and P_n is a "normalization" factor. In most computations it was 15 in order to make the effect of a parameter at the limit of its acceptable range comparable to a virtual-depth error of 15 km. The power n is any odd positive integer; generally it was 3 but occasionally values up to 9 were used. The purpose of n is to control the rate of increase of the "error" as the value of the particular parameter ventures outside the acceptable range.

The samples we shall discuss involve several computer runs, numbered arbitrarily. Each run took about a minute of computer time. These cases involve the use of the program to find an ionospheric model consistent with various observed points from a particular Alouette ionogram. The ionogram was made at $3^h\ 47^m\ 40^s$ UT on October 1, 1962. The sounder was at a height of 1038 km, located at $34^\circ.9$ N. latitude and $89^\circ.1$ W longitude. The geomagnetic dip angle was 67 degrees, and the electron gyrofrequency at the sounder was 0.94 Mc/s. We shall see that the results agree well with the true-height profile obtained from the same ionogram using the lamination method of Nelms [1963].

For the run which we have arbitrarily labeled #4, we chose to scale from the ionogram the virtual depths at those frequencies which would have been observed by the fixed-frequency topside sounder, 1964 51A (S-48). Only six virtual depths with corresponding frequencies and modes were used as input, and a mild constraint on the critical frequency was used. No constraint on the electron density at the sounder was used. Figure 1 displays the electron density profiles of the initial trial model and the final model. The model used in this case incorporated a "peak", i.e., it involved h_{max} as a parameter. Since the input data contain no information from which either h_{max} or the critical frequency could possibly be deduced, we made another run (run #5), this time simplifying the model by omitting the factor which involves h_{max}. The results of this run are shown in figure 2.

Agreement of the electron-density profiles is gratifying but only of secondary interest. Our chief concern is whether or not the method determines the parameters of the model uniquely and with acceptable accuracy. It is important that the final set of parameters be determined solely by the observed data and not by the particular initial parameters used for the trial model. This is the case when a satisfactory fit is obtained. An example is given in table 1 which summarizes the data illustrated in figures 1 and 2 and gives the parameter values that resulted.

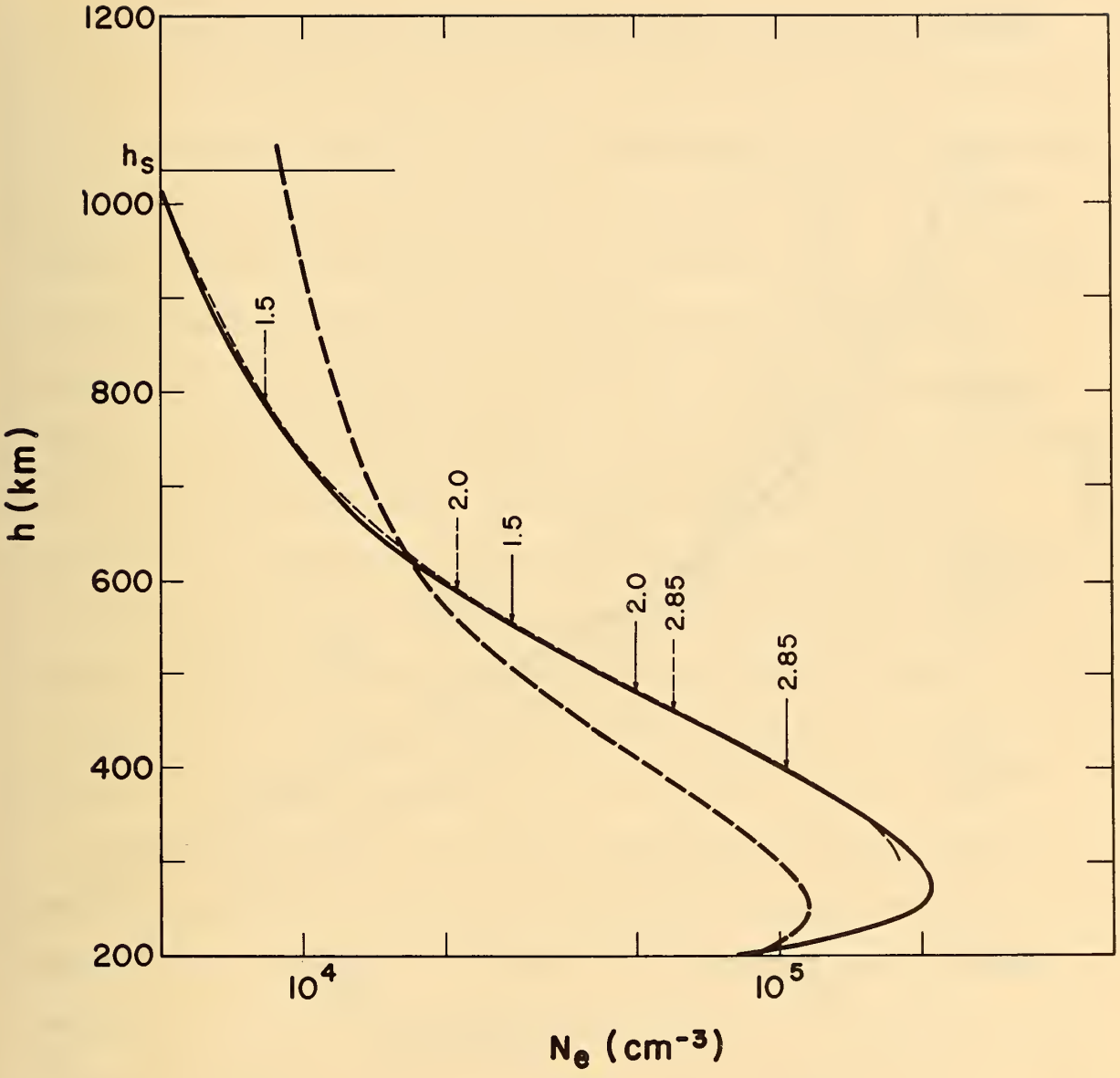

Figure 1. The electron-density profile (solid line) resulting from sample run #4, is compared with the profile (thin dashed line) calculated by a lamination method using 29 virtual depths scaled from the extraordinary trace of the same ionogram. The initial trial parameters produced the profile shown by the thick dashed line. Vertical arrows show the true heights of reflection, corresponding to the virtual depths of the final model, for the six pulses that were used. The final estimated rms error for run #4 of table 1 = 6 km.

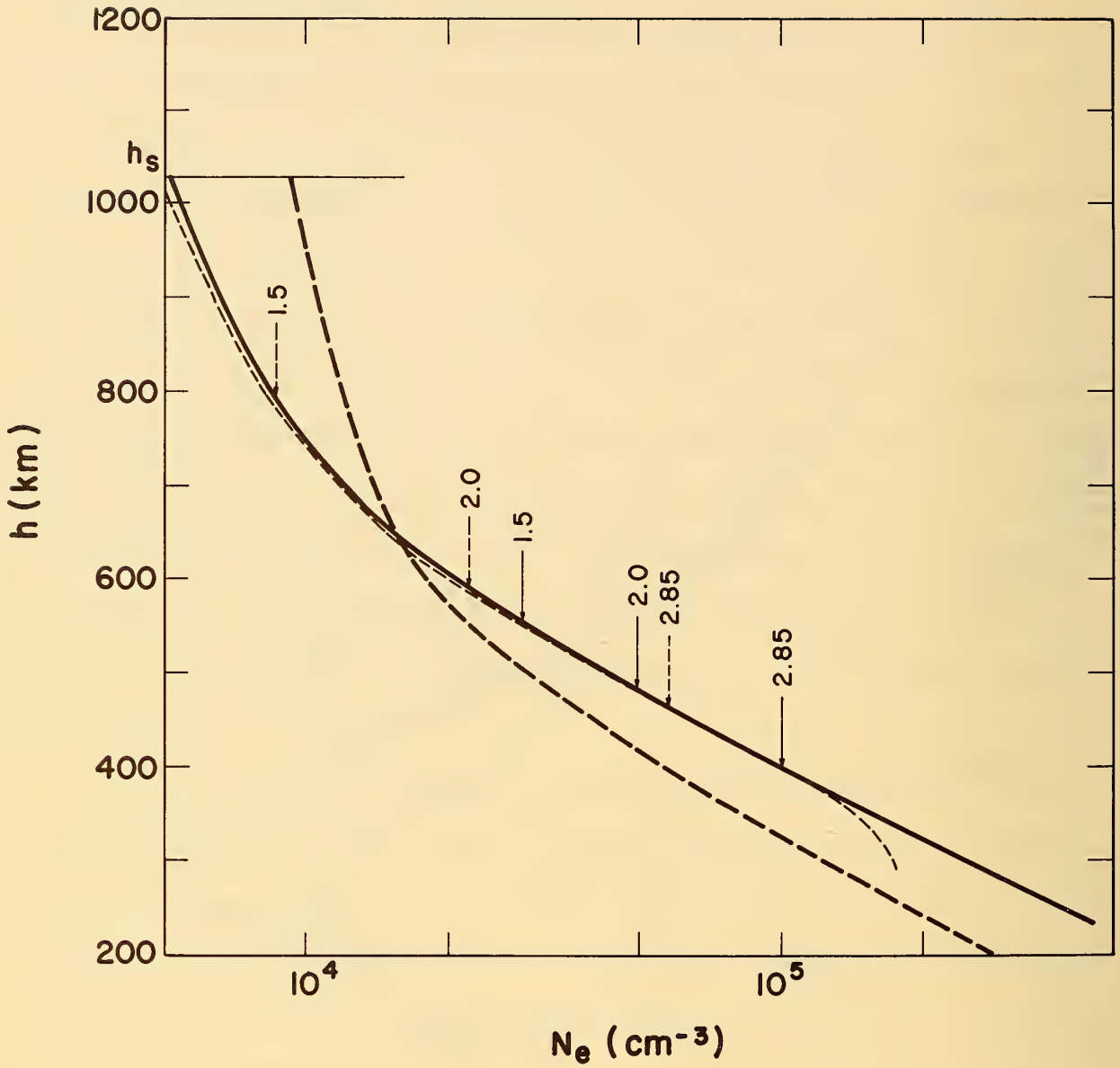

Figure 2. The result of sample run #5. The only difference from figure 1 is the omission of a peak in the model. Final estimated rms error for run #5 of table 1 = 8 km.

In reference to the question of uniqueness of the final parameters we made two runs using the same input frequencies and virtual depths but with different sets of initial values for the parameters. These two runs used six virtual depths scaled from the extraordinary trace of the ionogram. The plasma frequency at the satellite was given as a constraint, and the plasma frequency at a probable height of the maximum electron density obtained by the lamination method was also given as a mild constraint. Table 2 provides a summary of the input data and the results. The resulting electron-density profiles for each case are shown in figures 3 and 4, respectively. Both figures show the initial models, the final models and also, for comparison, the profile obtained by the lamination method. Also shown in each figure is the result of the preliminary adjustment of the parameters which was made in order to satisfy all bounds and constraints.

We made several other runs to test the use of the various types of information which would have been available from the fixed frequency sounder. Three of these (6, 9, and 10) are summarized in table 3. The same initial values for the parameters were used in all cases and no added constraints were incorporated except mild constraints on the critical frequency in 9 and 10. From the particular ionogram which we were using, it was possible to scale the virtual depths of three of the specified fixed frequencies: 1.5, 2.0, and 2.85 Mc/s. Both the extraordinary and the ordinary traces were present and were used for run #6. We made run #9 using only the 1.5 and the 2.0 Mc/s frequencies in order to investigate the capability of the method in dealing with very sketchy data.

The program is designed to use delay times of ground-reflected pulses as additional data. Unfortunately, the particular ionogram which we had selected for analysis showed no ground-reflected traces. Consequently, we calculated the virtual depths to be expected for the extraordinary and ordinary ground traces at 5.47 Mc/s and used this information together with the data of run #9 to make run #10 in our analysis of the simulated S-48 data. Results appear in table 3. The final parameters, virtual depths, and true heights agree well among the three different runs.

With the fixed-frequency sounder there exists a possibility of identifying the ordinary and the extraordinary pulse of a particular frequency incorrectly. In order to study the convergence behavior of the method in such cases, we made run #17 using both the ordinary and extraordinary modes of three S-48 frequencies but with the modes interchanged for the 2.0 Mc/s frequency. Table 4 summarizes the results of this run, and the corresponding run (#3) using the correct data. The results for the run using the incorrect data show differences of more than 100 km between the observed and computed virtual depths while the corresponding differences for the correct run do not exceed 7 km. The estimated rms error for the incorrect run was 107 km, which indicates the lack of a good fit as compared to 6 km for the correct run.

A summary of the final parameters and estimated rms errors of eight different runs is given in table 5. These runs utilize different minimization controls, different initial values of the parameters, various types of constraints such as the plasma frequency at the vehicle and h_{max}, and virtual depths scaled at different frequencies. Two of the runs used data from the electron-density profile determined by the lamination method. Thus, the computation of virtual depths was eliminated in these cases. Table 6 gives the averages and standard deviations of the final parameters for sixteen different types of runs, including those described in table 5, and also includes Rishbeth's estimate for comparison. We see that, for practical purposes, the results are quite insensitive to initial values of the parameters.

The programs have been used subsequently for about 150 ionograms. More than three-fourths of these produced excellent results; the remainder failed to fit the data within the estimated error of observation. Some of the failures exhibited residuals which indicated that more complicated models would be needed to fit the observations; five or six stubbornly refused to make any progress at all.

CONCLUSIONS

We have presented a method of analysis of topside ionograms which is particularly useful for sketchy or incomplete traces such as those from the fixed-frequency sounder. The method is a model-fitting method applicable to a wide variety of models and avoids the intermediate step of determining an explicit electron-density profile if only the parameters of the model are desired. The success of the method stems largely from the use of realistic bounds on the parameters and the ability to incorporate other constraints on the model. The method makes efficient use of input data, but requires more computing time than conventional true-height methods.

Tests with simulated data of the kind obtained from the fixed-frequency topside sounder show that several parameters of the ionosphere can be deduced from such records, even when it is impractical to produce complete electron-density profiles by conventional methods.

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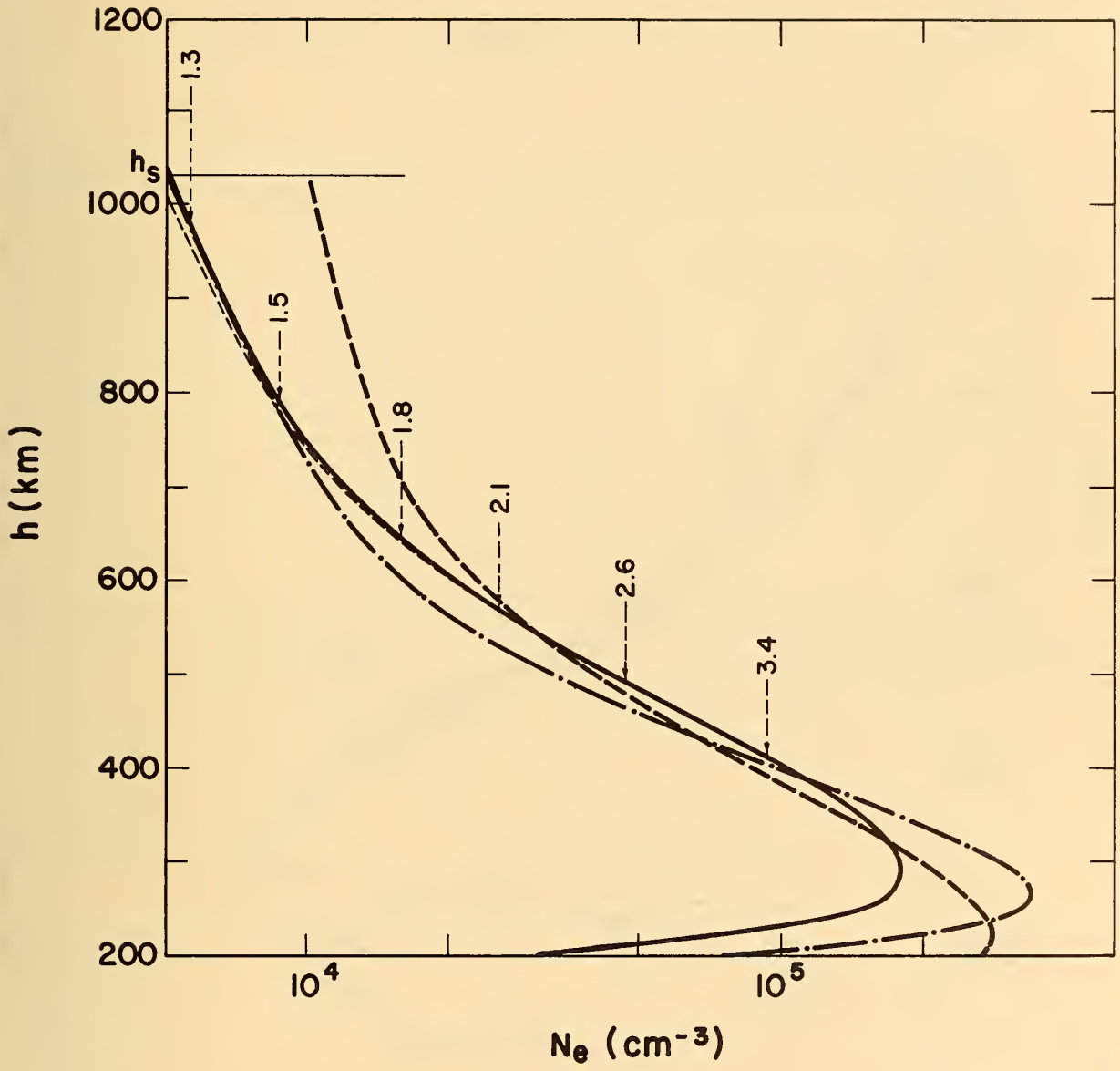

Figure 3. The electron-density profile (solid line) is compared with the profile (thick dashed line) produced by the initial trial parameters and the profile (dot-dashed line) obtained by preliminary adjustment to the bounds and constraints. The arrows show the true heights of reflection, corresponding to the virtual depths of the final model, for the six pulses that were used. The thin dashed line shows the profile produced by the lamination method. Final estimated rms error for run #1 of table 2b = 3 km.

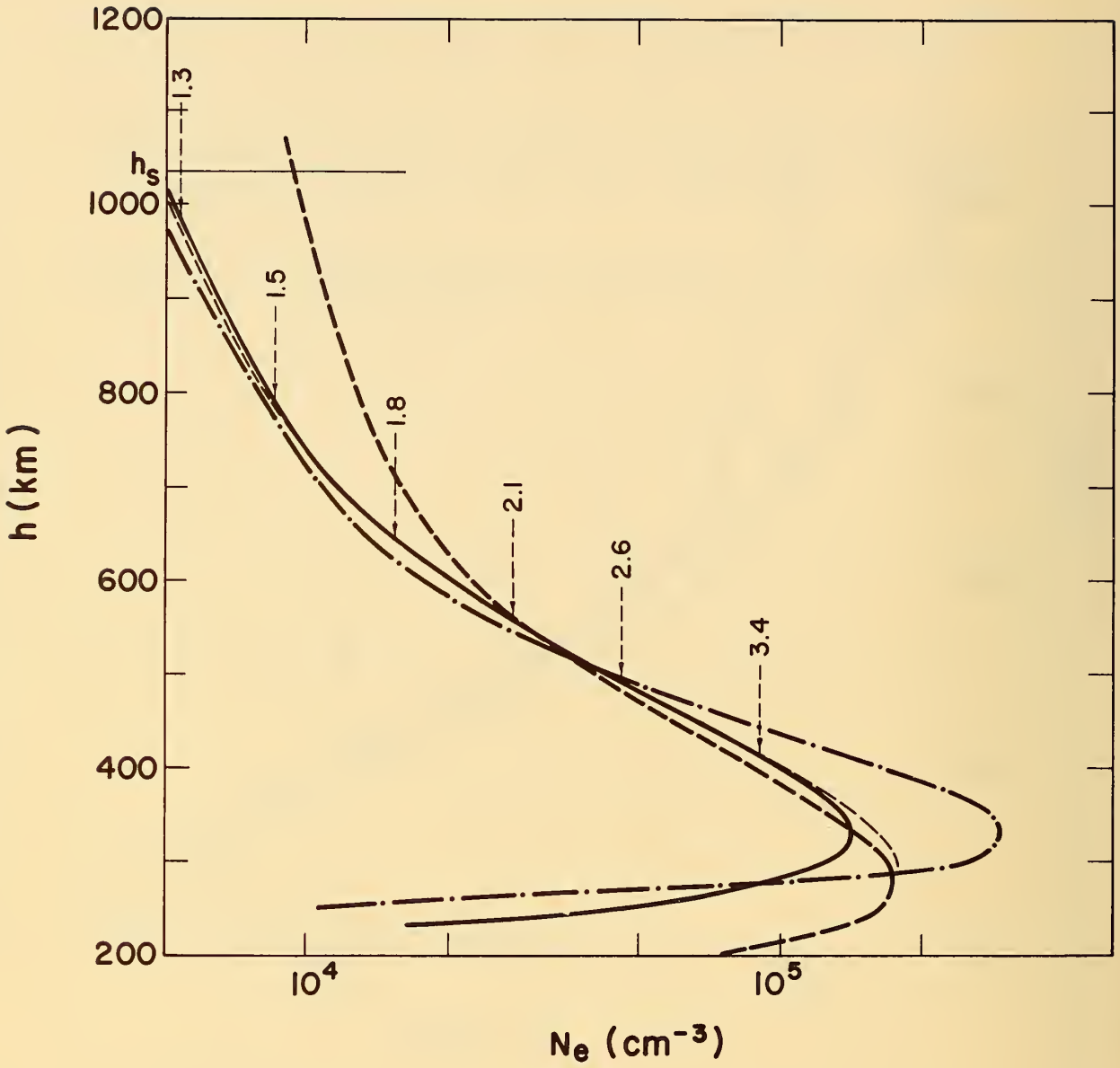

Figure 4. Same as figure 3 except that the model used different initial trial parameters. Final estimated rms error for run #2 of table 2b = 6 km.

Table 1:

INPUT			OUTPUT			
Frequency (Mc/s)	Mode	Virtual Depth (km)	Error in Virtual Depth (km)		True-height of Reflection (km)	
			Run 4	Run 5	Run 4	Run 5
1.5	X	1034	7	-2	785	792
2.0	X	878	-2	-1	588	593
2.85	X	838	4	8	461	462
1.5	O	713	-5	-3	551	556
2.0	O	744	5	6	480	482
2.85	O	825	-2	-10	397	399

Table 1a: Frequencies, virtual depths, and true-heights of reflection for the sample calculations illustrated in figures 1 and 2. The estimated rms error for run 4 was 6 km; the estimated rms error for run 5 was 8 km. Both estimated rms errors are based on n - k = 3 degrees of freedom.

Parameters	σ	h_{max} (km)	h_a (km)	N_a (cm^{-3} x 10^{-4})	T (°K)
Initial Model	.500	250	575	2.00	1000
Final Model Run 4	.053	274	651	1.43	878
Final Model Run 5	.047	---	666	1.40	928
Rishbeth's					
Estimated value	.05	---	650	----	840
Estimated Uncertainty	.10	---	10	----	50

Table 1b: The initial and final parameters of the models illustrated in figures 1 and 2. For comparison are given the values which Rishbeth (1963) deduced from the same ionogram by comparing a number of theoretical curves with the lamination profile.

Table 2:

INPUT			OUTPUT			
Frequency (Mc/s)	Mode	Virtual Depth (km)	Error in Virtual Depth (km)		True-height of Reflection (km)	
			Run 1	Run 2	Run 1	Run 2
1.3	X	600	-2	8	989	984
1.5	X	1034	3	2	795	792
1.8	X	930	3	6	647	643
2.1	X	861	-2	-4	573	570
2.6	X	833	1	-5	495	494
3.4	X	863	0	2	413	412

Table 2a: Frequencies, virtual depths, and true-heights of reflection for sample calculations illustrated in figures 3 and 4. Estimated rms error for run 1 was 3 km; estimated rms error for run 2 was 6 km. Both estimated rms errors are based on $n - k = 4$ degrees of freedom.

Parameters	σ	h_{max} (km)	h_a (km)	N_a ($cm^{-3} \times 10^{-4}$)	T (°K)
Initial Model Run 1	.500	225	600	2.25	900
Final Model Run 1	.029	294	648	1.54	872
Initial Model Run 2	.400	275	600	2.25	900
Final Model Run 2	.032	328	635	1.62	828

Table 2b: The initial and final parameters of the models illustrated in figures 3 and 4.

Table 3:

INPUT			OUPUT					
Frequency (Mc/s)	Mode	Virtual Depth (km)	Error in Virtual Depth (km)			True-height of Reflection (km)		
			Run 6	Run 9	Run 10	Run 6	Run 9	Run 10
1.5	X	1034	- 2	- 2	- 8	792	810	833
2.0	X	878	3	8	11	590	597	608
2.85	X	838	4	---	---	462	---	---
1.5	O	713	- 2	- 6	-10	553	560	570
2.0	O	744	3	0	- 6	481	487	496
2.85	O	825	-14	---	---	400	---	---
5.47	GX	1426	---	---	2	---	---	0
5.47	GO	1128	---	---	- 1	---	---	0

Table 3a: Frequencies, virtual depths and true-heights of reflection for sample S-48 data. Estimated rms error for run 6 was 11 km with n - k = 2, run 9 was 10 km with n - k = 1, and run 10 was 10 km with n - k = 3 degrees of freedom.

Parameters		σ	h_{max} (km)	h_a (km)	N_a ($cm^{-3} \times 10^{-4}$)	T (°K)
Run 6:		.050	251	652	1.48	896
Ordinary	1.5, 2.0, 2.85					
Extraordinary	1.5, 2.0, 2.85					
Run 9:		.106	261	653	1.55	884
Ordinary	1.5, 2.0					
Extraordinary	1.5, 2.0					
Run 10:		.061	260	646	1.73	883
Ordinary	1.5, 2.0					
Extraordinary	1.5, 2.0					
Ground Ord.	5.47					
Ground Ext.	5.47					

Table 3b: Final parameters for sample S-48 data.

Table 4:

INPUT				OUTPUT			
Frequency (Mc/s)	Mode		Virtual Depth (km)	Error in Virtual Depth (km)		True-height of Reflection (km)	
	Run 3	Run 17		Run 3	Run 17	Run 3	Run 17
1.5	X	X	1034	0	94	802	798
2.0	X	O	878	-2	-104	599	486
2.85	X	X	838	7	36	467	463
1.5	O	O	713	-5	-1	562	571
2.0	O	X	744	4	112	487	611
2.85	O	O	825	-4	31	401	388

Table 4: A test of the effect of mis-identifying ordinary and extraordinary echoes. The estimated rms error for run 3 was 6 km with $n - k = 3$, whereas 107 km was the estimated rms error for run 17 based on $n - k = 3$ degrees of freedom.

Table 5:

Run Number	σ	h_{max} (km)	h_a (km)	N_a ($cm^{-3} \times 10^{-4}$)	T (°K)	Estimated rms error
1	.029	294	648	1.54	872	2.6 km
2	.032	328	635	1.62	828	6.3 km
3	.078	243	673	1.41	929	6.3 km
4	.053	274	651	1.43	878	6.3 km
5	.047	---	666	1.40	928	8.4 km
6	.050	251	652	1.48	896	11.0 km
7	.111	282	626	1.88	899	.10 Mc/s
8	.109	283	631	1.82	899	.07 Mc/s
Mean of 8 runs	.064	279	648	1.57	891	
Standard Deviation of 8 runs	.030	26	15	.17	31	

Runs 1 and 2 used six extraordinary traces and the plasma frequency at the vehicle; runs 3,4,5, and 6 used the extraordinary and ordinary trace of three S-48 frequencies; and runs 7 and 8 used nine and twenty-nine points respectively of the lamination profile. Five different sets of initial parameters were used.

Table 6:

	σ	h_{max} (km)	h_a (km)	N_a ($cm^{-3} \times 10^{-4}$)	T (°K)
Mean of 16 runs	.094	277	650	1.55	833
Standard deviation of 16 runs	.061	29	19	.16	42
Rishbeth's Estimated value	.05	---	650	----	840
Estimated Uncertainty	.10	---	10	----	50

The sixteen different runs include the eight described in table 5. They use various data points, various initial sets of parameters, and a number of different constraints on the data.

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